A Brief Introduction To Mathematica

Dr. Christopher Moretti

Department of Mathematics

Southeastern Oklahoma State University

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Chapter One - Getting Started

Section One - Starting Mathematica

To launch the Mathematica application, open its folder and double-click the Mathematica icon. This will launch the portion of Mathematica known as the front end.

Mathematica is split up into two pieces - the front end and the kernel.

The front end is where you enter commands and where the results are displayed (be they plots, functions, whatever). The front end is similar to a word processor - it has no mathematical abilities itself, but it can open, save, and display Mathematica files (called notebooks). You do all of your work in the front end.

The kernel is where all of the mathematical computation is done. You never actually see the kernel at work (unless you open a special window to do so), even though that's where the real work is being done.

Mathematica is split up this way to conserve computer resources - especially computer memory. Mathematica's kernel is very powerful and can tax older computers. Since the front end is separate from the kernel, you can use the front end to enter things and view previous work without taxing the computer. When you double-click the Mathematica icon, it launches the front end. The kernel isn't launched until you try to evaluate your first expression.

Section Two - Input and Output

In the front end, information is grouped into "cell". A cell includes both the input and the output that is generated it by it. Cells are grouped together by square brackets on the right-hand side of the front end's window. To evaluate a cell (that is, have Mathematica process it), make sure your cursor is somewhere in the input portion of the cell (usually the topmost bracket in the grouping) and hit **shift-enter** (Windows) or **enter** (Macintosh). Throughout this text I will mention "entering" a command, or "using" or "running" a command. All of these mean putting the command into a cell and then evaluating the cell.

Once evaluated, each command you input (and you can have more than one in a cell) and its result are assigned numbers so you can refer to them later - the k-th command you enter is labeled In[k] and its output as Out[k]. For example, type 1+3 in a cell and evaluate it. You should see the input and output numbers on the left side of the front end's window. In a new cell, type the output number (if it was Out[2], then type Out[2] in the new cell) and then evaluate the cell. You should get 4 again, since Mathematica knows you're referring to the previous calculation. Mathematica does not go through all the work the second time, it just repeats the output - which can save a lot of time if the calculation you're skipping was complex. You don't have to just repeat the previous output, you can use it in a new calculation - if you enter 5*Out[2] and evaluate (assuming Out[2] was the number 4 from before), then Mathematica will return 20.

Another way to refer to previous input is through percent signs. Entering % in Mathematica is short-hand for "the last output you gave me". So if the last output was 8128, if you evaluate a cell with % in it you will get 8128. If you had entered %-5, you would get 8123. So the percent sign is a good way to quickly refer to your previous calculations without a lot of retyping (or a lot of cutting-and-pasting). You can even refer to outputs farther back by using more percent signs, like %% and %%%. If the last three outputs were $x^2 - 2x + 1$, 3x, and 5, then if the very next cell you evaluate is %%% Mathematica will return $x^2 - 2x + 1$. One thing you do need to be careful about is that the percent signs always work in chronological order. If you do 50 calculations in a notebook and then create a new cell all the way at the top (by clicking the mouse before the first cell and starting to type) with %%, the result will still be the 49th (second to last) output. Another thing to be careful of is that if you save a notebook and quit Mathematica, it forgets all of the previous input and output - both in the In[ ]/Out[ ] format and with percent signs. When you reopen the notebook you will have to go back and reevaluate the necessary cells the next time - and if you are using percent signs, you have to do it in the same order too. So if you are going to be saving your work to disk, retyping output or using copy and paste is not a bad idea.

There is another output trick which occasionally comes in handy. Sometimes you may want to do a calculation (maybe expanding $(x + 1)^{1000}$ or doing a complex 3-D plot) and not want to see the output because it would be too long or Mathematica is low on memory. If you end the input with a semi-colon Mathematica will process the input but won't print the output. You can still refer to it by Out[42] or %%% - it just won't be on the screen. So if you evaluate a cell with Expand[$(x + 1)^{1000} - x^{1000}$]; Mathematica will still process it, but the output (which would be several screens long) won't appear to clutter your work.

Section Three - Brackets and Braces

In Mathematica, different ways of grouping things have very different and specific notations.

Parentheses ( ) are used like we normally use them in expressions like 3*(2+1) or (x-2)(x+1).

Square brackets [ ] always belong to functions (built-in functions in Mathematica always start with capital letters, and sometimes have capital letters for each first letter in a compound word).

Curly brackets { } are used for defining lists and matrices.

Section Four - Exact and Approximate numbers and Built-in Constants

In Mathematica, all numbers are considered exact numbers unless you tell it otherwise. 2+3 is the exact number 5 - not 5.0 or 5.00. Even fractions like $\frac{1}{3}$ are considered exact.

If you wish to get a numerical estimate of a number, there are two ways to get it. The first is to follow the number by //N. This will get you the first 6 digits of the number - evaluating 1/3 //N gets you 0.333333. Another (more flexible) way of estimating a number (say 1/3) is to enter N[1/3]. This will get you 0.333333 again. However, if you want the first 20 digits of 1/3, you simply alter this to N[1/3,20]. If you wanted the first thousand digits, you would enter N[1/3, 1000].

Mathematica has a number of built-in constants - numbers which come up so often they get their own notation. Like virtually all built-in objects they start with capital letters. Here are a few of the more important ones.

Pi - the number $\pi \approx 3.14159$

E - Euler's number $e \approx 2.71828$

I - the imaginary number $i = \sqrt{-1}$

Degree - the number of radians in one degree $(\frac{\pi}{180})$

Infinity - ∞.

So if you wanted the first 10000 digits of π, you would enter N[Pi,10000].

Section Five - Loading Additional Packages

There are many packages of additional functions for Mathematica - add-ons which cover functions and areas not built into the main program. These can be loaded in if you know the name of the package and the file path the package is in. For example, to load the information contained in the package ImplicitPlot under the heading Graphics, you would enter <<Graphics`ImplicitPlot`(you must use the accent sign ` when loading a package - the open quote ‘ won’t work). The << tells Mathematica to load a package, Graphics is the file path and ImplicitPlot is the name of the package. After evaluating this command, you can now use the commands defined in the package as if it were part of the program from the start. If you quit and restart Mathematica, you will have to reload the package (and any others you may have loaded).

Chapter One Exercises

1) Use Mathematica and numerical approximation to find which is bigger: e^{π} or π^{e}. Powers in Mathematica are entered as the ˆ symbol.

2) Using Mathematica, find $i^2, i^3, i^4, i^5,$ and i^6.

3) What are the first 15 digits of e?

4) Load in the the packages ImplicitPlot and Polyhedra, which can both be found in the file path Graphics.

5) Try to load in the package ThisIsNotARealPackage from the file path Algebra (which is a real file path with several packages for extending Mathematica's algebraic abilities).

Chapter Two - Mathematical Functions

Section One - Built-In Functions

Mathematica has several thousand common mathematical functions built into it covering everything from elementary algebra to advanced functions from real analysis and number theory. Here is a list of common functions and their notations. If you ever want information on a function, simply enter a question mark before its name and evaluate that cell. For example, if I wanted to know what the function Cos[] is for, I would evaluate ?Cos. If I wanted technical information (like special options), I would evaluate ??Cos. If you are not sure of the name of a function, you can use a * symbol as a "wild card". If I wanted to know the names of all functions starting with Cos, I would enter ?Cos*. You can even use more than one wildcard - ?C*s* gives the names of all functions starting with C with an s in them. Throughout this text I will refer to functions by putting [] after their name, like Cos[] or Plot[]. When you use the functions in Mathematica you will have to fill in the appropriate information between the brackets (like Cos[x] or Cos[3Pi]) - this is just some shorthand which will make it easy to refer to functions. All functions use square brackets around their arguments.

Arithmetic Notation

Addition: Use the + symbol.

Subtraction: Use the - symbol.

Multiplication: Use either * or a space. Be careful when using spaces - Mathematica interprets xy not as x*y but a single variable whose name is "xy".

Division: Use the / symbol.

Exponents: Use the ˆ symbol (2^3 = 2ˆ3). For fractional exponents, use parentheses - for $81^{1/4}$, evaluate 81ˆ(1/4).

Absolute Value: Abs[x] is the absolute value of x. This works for complex numbers as well as real numbers.

Sign[x]: The sign of x: 1 if x>0, -1 if x<0, 0 if x=0.

Square roots: Sqrt[x] is the square root of x.

Factorials: n! is the factorial of n (3! will give you 6, 7! will give 5040). If you inadvertently try to take the factorial of something which is not a whole number, Mathematica will return an expression involving the Gamma[ ] function, which extends the factorial idea to non-whole numbers.

Binomials: Binomial[n,k] gives the binomial coefficient $\binom{n}{k}$, the number of subsets of k objects from a set of n objects.

One thing you need to be careful of is taking odd roots of negative numbers. If you enter (-8)^(1/3) in Mathematica, it will return a complex root of -8 rather than -2. You can get around this using Sign[ ] and Abs[ ] as Sign[-8] Abs[-8]^(1/3).

Trigonometric Functions

In Mathematica, all trigonometric functions and their inverses use radian measure for angles. So if you want to find the sine of 180 degrees you will need to include the conversion factor Degree as Sin[180 Degree]. Remember, Mathematica will return exact numbers if exact numbers are given as input - so Sin[2] is simply Sin[2] unless you numerically approximate it. Mathematica knows how to use complex numbers with trigonometric functions - so if you inadvertently enter something which you think of as undefined - like the inverse sine of 2 - you may get a complex answer (you will usually need to numerically approximate to see this).

Sin[x]: The sine of x	ArcSin[x]: The inverse sine of x.
Cos[x]: The cosine of x	ArcCos[x]: The inverse cosine of x.
Tan[x]: The tangent of x	ArcTan[x]: The inverse tangent of x.
Cot[x]: The cotangent of x	ArcCot[x]: The inverse cotangent of x.
Sec[x]: The secant of x	ArcSec[x]: The inverse secant of x.
Csc[x]: The cosecant of x	ArcCsc[x]: The inverse cosecant of x.

Exponential and Logarithmic Functions

Again, Mathematica can process complex numbers, so you may see unexpected things if you try to do things like take logarithms of negative numbers.

Exp[x]: The natural exponential function e^x (this can also be obtained by E^x).

Log[x]: The **natural** logarithm of x. This differs from standard notation ln(x).

Log[b,x]: the logarithm of x in the base b. So the "common logarithm" of x is Log[10,x].

Hyperbolic Functions

The hyperbolic functions as their inverses are defined in terms of exponentials and logarithms - so complex numbers are valid arguments and may appear as output from these (if you try to find the inverse hyperbolic cosine of 0, for example) - although you may need to numerically approximate the values to see this.

Sinh[x]: The hyperbolic sine of x
Cosh[x]: The hyperbolic cosine of x
Tanh[x]: The hyperbolic tangent of x
Coth[x]: The hyperbolic cotangent of x
Sech[x]: The hyperbolic secant of x
Csch[x]: The hyperbolic cosecant of x

ArcSinh[x]: The inverse hyperbolic sine of x.
ArcCosh[x]: The inverse hyperbolic cosine of x.
ArcTanh[x]: The inverse hyperbolic tangent of x.
ArcCoth[x]: The inverse hyperbolic cotangent of x.
ArcSech[x]: The inverse hyperbolic secant of x.
ArcCsch[x]: The inverse hyperbolic cosecant of x.

Section Two - Defining Functions and Equalities

When working with Mathematica, it is often useful to define your own functions, like f(x)=$x^2-\sqrt{x}e^{\sin(2x)}$. Mathematica has a special notation for equalities which define functions. To define the function f(x) above, you would enter

f[x_]:=x^2-Sqrt[x] Exp[Sin[2x]]

The special equality symbol := is used for defining functions (think of it as "let what comes before this symbol be"). When you define functions, you must use square brackets. Also notice the underscore following the x on the right - this means that whatever you enter for x is plugged into the formula on the right-hand side (whether it is a number or some expression involving variables like t+1). Once you have evaluated this, you can then evaluate expressions like f[2], f[Cos[t]], f[x+h], or even f[Bob]. If your function is a pure numerical function like f(x)=x^2, then you can omit the : in the function definition - f[x_]=x^2 will work just fine (and it will print out the definition of your function so you can check it). However, if you want to include a Mathematica command as part of your function - like Expand[x^2] for example - you must include the : in the definition (f[x_]:=Expand[x^2]). Omitting the : in this case won't define the right function - Mathematica will still square numbers, but won't "Expand" inputs like (x+h).

You can define functions of more than one variable in the same way. To define g(x,y)=$xy-\sin(x\ \ln(y))$, you would evaluate g[x_,y_]:=x y-Sin[x Log[y]]. Notice the space between x and y in the first term and between x and Log[y] in the second - both stand for multiplication. We can now plug numbers into this function (say x=1 and y=e/2) by evaluating g[1,E/2].

At some point you may wish to redefine a function. While usually just redefining a function will work, once you get into more complicated functions and programming this may cause some problems (parts of the old definition may conflict with parts of the new one). The best way to do this is to remove the old definitions before entering the new one. This can be done with the Clear[ ] command. To eliminate the definition for g(x,y) above, you would evaluate Clear[g]. This command removes all definitions for g, so this eliminates the possibility of conflicting definitions.

It is tempting to drop the brackets and underscores and just enter something like f=xˆ2 for the function f(x)=x^2. This means something different than defining a function in Mathematica. You have now defined f to be the "object" $x^2 - 1$ - that is, f is now a nickname for the expression $x^2 - 1$. You cannot plug numbers into this (at least not directly). You can do other things to it - you could construct a plot of f as x goes from -3 to 4, take the derivative of f, and try to factor f. But it's no longer a function - f just becomes a convenient way to refer to $x^2 - 1$. This nicknaming can be undone just like function definitions by entering Clear[f].

Another thing to watch out for is the notations of equations in algebra. When we have an equation like 2x+3y=5 in algebra (defining a line, for example), this equation can be either true or false depending on what values of x and y you plug in (it's true when x=1 and y=1 but false if both are 0). This type of equation (one which is true or false rather than referring to the definition of a function or a nickname) has a different symbol in Mathematica. You would represent this true/false type of equation with ==. The equation above would be entered as 2x+3y==5. This comes up with the Solve[ ] command which is used to find solutions to equations. Entering 1==2 yields the output False, whereas 4==2ˆ2 gives True.

Chapter Two Exercises

1) What is $\frac{256^2-8128}{40}$?

2) Have Mathematica simplify $\sqrt{964}$ (it will do this automatically when you evaluate the expression for $\sqrt{964}$).

3) Have Mathematica find $|\frac{2^4-3^3}{6^7-7^6}|$.

4) What is 20! ?

5) How many ways are there to pick four objects out of 20?

6) Numerically estimate Sin[1] and Sin[90]. Why isn't Sin[90]=1?

7) What is cos(i ln(2))?

8) What is $\cos^{-1}(\frac{1}{2})$? Numerically estimate $\sin^{-1}(\frac{3}{5})$.

9) What is $\tan(\sin^{-1}(1/7))$? Find this exactly and estimate the value.

10) Estimate ln(3) and $\log_{10}(3)$.

11) Estimate sinh(1) and cosh(1).

12) Define the function f(x)=$\frac{\sin(x)}{\cos(x)+3}$.

13) Define g(x)=$x^{\frac{1}{\ln(x)}}$. What are g(2) and g(3)?

14) Define h(x,y)=x^2-sin(xy). What is h(0,3)?

15) Having defined the functions f, g, and h in the previous 3 problems, have Mathematica show you the definitions of them using the ? command.

16) Clear the functions f, g, and h from Mathematica's memory.

17) Explain the differences in how Mathematica interprets the commands k[x_]:=3x-4 and k=3x-4.

18) Explain the differences in how Mathematica interprets x=3 and x==3.

19) Define a function MyCubeRoot[x] which will give the usual values for cube roots if x is positive, negative, and zero.

Chapter Three - Lists and Matrices

Section One - Lists

A list is just an ordered set of objects. Lists always use the curly braces {}. {1,2,3}, {x,Sin[x],Log[x],x}, and {Out[3], Piˆ2,2+3I} are all lists. Objects can be repeated in a list, and since ordering makes a difference the set {1,2} is different than the set {2,1} (this can be checked by evaluating {1,2}=={2,1} which will yield False). You can even have lists of lists like { {1,2}, {1,2,3}, {-3,2,x} }.

Section Two - Generating Lists

Lists can be entered like any other object. If you need to refer to {1,2,3,4,5,6} over and over, you would just enter f={1,2,3,4,5,6} and then you could refer to the list by the nickname f.

If you need to generate a list from a formula (in the variable i) from integer values *i-start* to *i-finish* , use the Table[ ] command

$$\text{Table[formula,}\{i, i - start, i - finish\}]$$

Table[iˆ2,{i,0,3}] returns {0,1,4,9}, and Table[xˆi,{i,4,7}] returns {x^4,x^5,x^6,x^7}.

There's no reason to have your formulas jump up by 1 each time - you can go up by any increment *di* with the command

$$\text{Table[formula,}\{i, i - start, i - finish, di\}]$$

Table[i xˆi,{i,-1,5,2}] returns the list $\{-\frac{1}{x}, x, 3x^3, 5x^5\}$. If the value for *di* is negative, then the Table[ ] command will create a list which goes down from *i-start* to *i-finish* - for example
Table[2i,{i,5,3,-1}] will return {10,8,6}. If you are going to use a negative value for *di* , make sure that the value *i-start* is bigger than the value *i-finish* .

A shortcut for generating the list {1,2,3. . . ,n} is Range[n]. For example, Range[5] yields {1,2,3,4,5}.

If your formula involves more than one variable you can create lists of lists using

$$\text{Table[formula,}\{i, i - start, i - finish, di\}, \{j, j - start, j-, finish, dj\}].$$

If you omit the *di* or *dj* parts (including the comma) Mathematica assumes they are 1, just as if there were only one variable.

The way your sublists are ordered is: the first sublist corresponds to i=*i-start* and j goes from *j-start* to *j-finish* ; the second sublist is the list i=*istart+di* and j goes from *j-start* to *j-finish* , and so on. The command Table[x^i+y^j,{i,1,3},{j,1,2}] yields $\{\{x+y, x+y^2\}, \{x^2+y, x^2+y^2\}, \{x^3+y, x^3+y^2\}\}$. This is the best way to create matrices - the command Table[a[i,j],{i,1,m},{j,1,n}] returns an m-by-n matrix whose entries are a[i,j] (ideally, you have previously defined what a[i,j] means by something like a[i_,j_]:= -your formula in i and j here -).

You can pick elements out of lists that have already been defined. If f is a list, then f[[i]] is the i-th element of f. So if f={1,2,3,4,7}, f[[1]] is 1 and f[[5]] is 7. If g={{1,2},{1,2,3}}, then g[[1]] is the list {1,2}. If f is a list of lists, you can pick out individual elements of f; f[[i,j]] is the j-th element from the i-th list in f. So in the list g above, g[[1,1]] is 1, g[[1,2]] is 2, and g[[2,3]] is 3.

Section Three - Functions Involving Lists

Lists come up quite a bit in Mathematica, so there are many important functions for manipulating them. In the following, let *list1* and *list2* represent the names of two previously defined lists.

Length[*list1*]: The number of objects in *list1*. Sublists count as single objects, so
Length[{{1,2,4},{3,2,1}}] is 2 (there are two things in the list even though the objects themselves are lists).

Dimensions[*list1*]: The dimensions of *list1*. Dimensions[{{1,2,4},{3,2,1}}] will return {2,3} as this is a 2-by-3 list (a list of 2 objects both of which have length 3).

Union[*list1*,*list2*]: The union of *list1* and *list2*. Repetitions will be deleted. Union[*list1*] will return a list made from *list1* with repetitions deleted.

Intersection[*list1*,*list2*]: A list which is the intersection of *list1* and *list2* (the objects common to both).

Flatten[*list1*]: This creates a new list from list1 by “flattening” out all sublists of *list1*. Flatten[{{1,2,3}, {1,2,3,4}}] returns {1,2,3,1,2,3,4}.

Section Four - Displaying Lists and Matrices

After defining a list (especially lists of lists), it is useful to be able to see the list displayed as an array. This can be done with the commands TableForm[*listname*] and MatrixForm[*listname*]. For example, if we had defined m to be the list { {1,3,2},{4,6,5}}, TableForm[m] would return the array

$$\begin{matrix} 1 & 3 & 2 \\ 4 & 6 & 5 \end{matrix}$$

MatrixForm[m] places brackets around the array to conform to standard notation for matrices - MatrixForm[m] will return

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \end{bmatrix}$$

Section Five - Plugging Into Expressions

Often you may want to plug a number or set of numbers into some expressions without formally defining it as a function. If you need to evaluate $3x^5 - 15x + 5$ at x=1 and nothing else, it's probably overkill to define f[x_]:=3x^5-15x+5 and then evaluate f[1]. Mathematica has a notation for plugging into an expression directly without defining it as a function. To replace the variable x in the expression f with the value a, evaluate f /. x->a (the arrow is composed of the - symbol followed by >). So if you evaluate x^2 /. x->3, Mathematica will return 9. If you evaluate x^2-x /. x->(t+1), Mathematica will return $(t + 1)^2 - (t + 1)$. Giving a value for x with the /. notation is called using a "replacement rule". Replacement rules are temporary - in the last example no value has been assigned to $x^2 - x$; Mathematica has simply told you what it obtains when x is replaced by t+1.

If an expression has more than one variable in it, you can do multiple replacements using lists. For example, if you enter x^2-y /. {x->3,y->Sin[t]}, Mathematica will return 9-Sin[t]. You can substitute in as many variables as you want using lists, not just 2, so the list {x->1,y->3,z->-6,w->16t^(-3)} is a valid replacement list.

If you want to plug in several different things in for x into the same expression, you can modify this replacement rule idea slightly. If you want to see a list of the values of x^2 when x is 0,2,3, and 9, evaluate x^2 /. x->{ {x->0}, {x->2}, {x->3}, {x->9} } (notice each value you want to plug in has its own replacement rule in a sublist). The result of this will be the list {0,4,9,81}. You can even use this in conjunction with the Table[] command. If you wanted to get a list of the values of x^2 as x takes on the numbers -5,-4,...,4,5, you could evaluate the command xˆ2/.Table[{x->i},{i,-5,5}]. Again, notice the {} brackets around the rule x->i in the Table[] command - each little rule has to be in its own sublist.

Many important functions like Plot[] and Plot3D[] have various options which are entered using the -> notation. See Chapters Eight and Nine for examples of options given by the arrow notation.

Chapter Three Exercises

1) Create a list of the numbers 1,2,...,30.

2) Make a list of the first 30 odd numbers.

3) Define a to be the list {1,2,-3,x,5,y}. Using the [[]] notation, have Mathematica find the third and sixth elements of a.

4) Create a list of numbers of the form 2^i as i goes from -10 to 20 by fives.

5) What is the result of the command Table[i/j,{i,1,4},{j,1,5}]?

6) Find the "length" and "dimensions" of the list from problem 5.

7) Create a list from {1,2,3,1,2} by removing the duplicates.

8) What numbers are common to the first 100 multiples of 4 and the first 75 multiples of 5?

9) Enter the matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 2 \end{bmatrix}$ into Mathematica. Have Mathematica display this as a matrix.

10) Evaluate the polynomial $3x^5 - 15x - 5$ at x=1, x=2, and x=3.

11) Find the value of $\sin(xy) - x^2y$ at the point $(\frac{\pi}{4}, 2)$.

12) Find the values of $x^4 - x^3$ at x=-1,0,1,2,3,4,5 using a single command.

13) In the expression $x^2 + 2xy - y^2 - 6x + 4y + 1$, have Mathematica replace x with x1+h and y with y1+k.

Chapter Four - Mathematica and Algebra

Mathematica's representation of polynomials is the opposite of the one we usually use - it lists them from lowest-degree term to highest. So if you enter xˆ3+x+1, it will return this as $1+x+x^3$.

Mathematica has many functions for performing algebraic calculations. Some of the more common ones are (let *expr* be a generic expression, not necessarily in just one variable):

Expand[*expr*]: Expand *expr* by performing all multiplications. Expand[(x+1)ˆ3] will return $1+3x+3x^2+x^3$. Expand[(x-y)ˆ2] will result in $x^2-2xy+y^2$.

Factor[*expr*]: Attempt to factor *expr*. Mathematica will attempt to factor polynomials using whole number coefficients. Factor[2xˆ2-2yˆ2] returns 2(x-y)(x+y). Factor[xˆ2-2] returns -2+x^2, as this cannot be factored using integer coefficients.

FactorInteger[*expr*]: If expr is an integer, FactorInteger[*expr*] will return a list of the prime factors of expr and their multiplicities. FactorInteger[40] will return { {2,3},{5,1}} as in the factorization of 40 into primes 2 appears 3 times and 5 appears once.

Collect[*expr*,x]: Write *expr* as a sum of powers of x. Collect[2xˆ2+3 x y+2x+1,x] will return $1+2x^2+x(2+3y)$. This will be different than what you will get if you evaluate the command Collect[2xˆ2+3 x y+2x+1,y], since this will collect the coefficients of the powers of y rather than x.

Simplify[*expr*]: Write *expr* as simply as possible. Mathematica's notions of "simple" may not always match yours. Simplify[2(x+10)-5(3x+4)] will return -13x.

Together[*expr*]: Bring all the pieces of *expr* together over a common denominator.
Together[1/x-1/(xˆ2-1)] will return $\frac{-1-x+x^2}{x(-1+x^2)}$.

Apart[*expr*]: Break apart *expr* as the sum of fractions with denominators as simple as possible. Apart[(-1-x+xˆ2)/(x(-1+xˆ2)] will return $-\frac{1}{2(-1+x)}+\frac{1}{x}+\frac{1}{2(1+x)}$.

Cancel[*expr*]: Cancel any common factors in *expr* . Cancel[(xˆ2-x)/(x-1)] will return x.

PolynomialDivision[*poly1*,*poly2*,x]: Divide the polynomial *poly2* into *poly1* , treating x as the variable. It returns the list {quotient, remainder }. PolynomialDivision[xˆ5,xˆ2-x+1,x] returns {-1+x^2+x^3,1-x} as x^2-x+1 goes into x^5 (x^3+x^2-1) times with a remainder of $-x+1$.

Coefficient[*expr*,x]: Find the coefficient of x in *expr*. Coefficient[3x y+2xˆ2-6x+1,x] will return 3y-6.

Chapter Four Exercises

1) Have Mathematica simplify the expression $\frac{(x+h)^4-x^4}{h}$, and then use it to substitute in h=0.

2) Use Mathematica to expand what you get when you replace x with x1+h and y with y1+k in the expression $x^2+2xy-y^2-6x+4y+1$. Then have Mathematica find the coefficient of (x1)(y1), x1, and y1 in the expansion.

3) Have Mathematica decompose $\frac{x^{12}}{x^8-1}$ into simpler rational functions. Then have it bring those fractions back together again.

4) Factor $x^{20}-1$.

5) What are the prime factors of $2^{126}-1$?

6) What are the quotient and remainder you obtain when you divide x^3-1 into x^6-x^4+7?

7) Cancel all common factors in $\frac{x^{12}-1}{x^9-1}$.

8) Simplify $(x+2)^3-(x-1)^2+(x+9)^4-1$.

Chapter Five - Solving Equations

Mathematica does a very good job of solving equations, although there are still limits to what it can do. The basic command for solving equations is Solve[*equation*, *variable*]. If we wanted to solve $x^2 + 3x + 2 = 0$ for x, we would use the command Solve[xˆ2+3x+2==0,x] (remember, the == notation must be used for these types of equations). The result from this command is the list {{x->-2}, {x->-1}}. The solution is given as a set of replacement rules. If you type x /. % immediately after the Solve[ ] command, we would get the list of solutions {-2,-1}. You can even have Mathematica solve for x in an equation with more variables. The command x /. Solve[2x+3y==1,x] will return the solution list $\{\frac{1}{2}(1 - 3y)\}$ (Solve[...] returns a set of replacement rules so x /. Solve[...] makes good sense, as it is "x replaced using the rules for replacing x that Solve[...] returns" - See Chapter Three, Section Five for a discussion of replacement rules). Solve[a xˆ2+b x+c==0,x] will give you the replacement rules which correspond to the quadratic formula. If an equation has no solution (or if Mathematica cannot find one), Mathematica will return the empty list {}.

Even when dealing with a single variable, Mathematica can only do so much when solving equations. While it can solve any polynomial equation in x of degree 4 or less exactly (like $x^4 - 3x^2 - 2x - 1 = 0$), it may or may not be able to solve equations of higher degree (there is no formula for roots of polynomials of degree 5 or more using elementary functions). For example, Mathematica cannot solve $3x^5 - 15x + 5 = 0$. In this case it returns an abstract representation of solutions using a command called Root[ ]. If Solve[ ] fails, try to numerically estimate the solutions using NSolve[ ], which uses exactly the same format as Solve[ ] and gives solutions as replacements rules like Solve[ ] does. NSolve[3xˆ5-15x+5==0,x] will return replacement rules for approximations of the 5 roots, including a complex conjugate pair.

Mathematica also has trouble solving equations involving more complicated functions - sometimes even if it can find some solutions it may miss others. It will usually warn you this is happening with an error message which mentions "transcendental functions" or "inverse functions". It cannot solve sin(x)=x, for example, even though x=0 is an easy solution. It can solve sin(x)=1 - but it will only catch the solution x=$\frac{\pi}{2}$ and miss all of the others. In other cases, it may return solutions using unfamiliar functions like ProductLog[ ] - remember, you can always investigate such functions using the ? for help.

Mathematica can solve systems of equations as well, using the format Solve[*equation-list*, *variable-list*]. For example, to solve the system of equations 2x+y=1, 3x-y=0, use the command Solve[{2x+y==1,3x-y==0},{x,y}]. This will return a list of replacement rules again, in this case {{x-> $\frac{1}{5}$,y-> $\frac{3}{5}$}}. If you wanted a list of the solutions rather than replacement rules, you could enter {x,y}/.Solve[{2x+y==1,3x-y==0},{x,y}] just as for a single equation.

Mathematica can handle more than just 2 equations in 2 variables - it is limited only by the available memory and your patience in typing the list of equations in.

Chapter Five Exercises

1) Solve the system of equations 3x+2y=1, 6x-y=10.

2) Solve the system of equations 3x+2y=1, 6x+4y=5.

3) Solve the system of equations 3x+2y=1, 9x+6y=3.

4) Solve the system of equations 3x+3y-z=1, x-3y+2z=0, x-4y-z=-3.

5) Find the point(s) of intersection of the circle $x^2 + y^2 = 4$ and the line y=3x+1.

6) Solve the equation $x^2 + 2x - 6 = 0$.

7) Solve the equation $x^2 + 4x + 10 = 0$.

8) Solve the equation $x^3 - 4x + 1 = 0$.

9) Solve the equation $x^4 + 3x^2 - 6x + 2 = 0$.

10) Numerically estimate the solutions to problem 9.

11) Numerically estimate the solutions to $3x^5 - 15x - 5 = 0$.

12) Try to find solutions to x=10ln(x). Numerically estimate the results.

13) Solve the equation $x^3 - x^2 - 4x + 4 = 0$ and substitute the solutions into $x^2 - 1$.

Chapter Six - Calculus

Section One - Limits

Mathematica can calculate limits, including many of those involving infinity. To have Mathematica find the limit of f(x) as x approaches a, evaluate Limit[f[x],x->a] (here you'd have to previously define f[x] or just put in the formula for it). To find the limit of x^3 as x approaches 4, evaluate Limit[xˆ3,x->4]. If a limit does not exist at all and Mathematica detects this it will tell you. Infinity and -Infinity are valid points to "approach" in a limit, so Limit[xˆ(-2),x->0] can yield Infinity and you can take limits like Limit[(xˆ2-x-3)/(3xˆ2+2x+5),x->Infinity] (which will return 1/3). Mathematica can also handle more complicated indeterminate forms. For example, entering Limit[x-Sqrt[xˆ2-9],x->Infinity] will return 0, and Limit[(1+1/x)ˆx,x->Infinity] will return the constant E.

Section Two - Derivatives

Mathematica can take derivatives as well. To take the derivative of some expression *expr* with respect to x, use the command D[*expr*,x]. D[xˆ2,x] will return 2x - but D[xˆ2,y] will return 0, since x^2 is a constant with respect to y.

It is often useful to use the Simplify[ ] command in conjunction with the D[ ] command to try to simplify the result of the derivative. For example, D[Log[1+x]-Log[1-x],x] will return $\frac{1}{1-x}+\frac{1}{1+x}$, but Simplify[D[Log[1+x]-Log[1-x],x]] will return $-\frac{2}{-1+x^2}$ which is a bit easier to work with. When wrapping commands like Simplify[ ] around other commands, take care to make sure you balance the square brackets - if you leave a bracket off (or put too many in), Mathematica will return an error message warning you about it. If Mathematica returns a derivative which is unexpectedly complicated, you can always follow the derivative command with Simplify[%].

You can take higher-order derivatives in different ways. If your f is some previously defined expression (say f=xˆ5-5x), you can take the second derivative of f as either D[D[f[x],x],x] (literally the derivative of the derivative of f), more simply as D[f,x,x] (two derivatives with respect to x), or D[f,{x,2}]. If you wanted to take the fourth derivative of f with respect to x, the best ways to do this would be as D[f,x,x,x,x] or D[f,{x,4}]. In general, D[f,{x,n}] will return $\frac{\mathrm{d}^n\mathrm{f}}{\mathrm{dx}^n}$.

If you take the time to define a function like f[x_]:=3x^5-15x+5, you can also take derivatives using the standard "prime" notation. f'[x] will be the first derivative, f"[x] the second derivative, and so on. Each additional derivative requires another single-quote mark.

The D[ ] command is useful in combination with Solve[ ]. For example, to find the local extreme values of a function f(x) whose derivative is continuous, you find the derivative, set it equal to 0, solve for x, plug these numbers into the second derivative to check if they are maxs or mins, and then plug the numbers back into f(x) to get the actual values. You can do this for f(x)=$3x^5 - 15x + 5$ as follows:

First define f[x_]:=3x^5-15x+5. Then evaluate the command

{ {x,f[x]},f'[x],f"[x] } /. Solve[f'[x]==0,x].

The list on the left is the list { (x-coordinate of point,y-coordinate of point), derivative at point, second derivative at point } replaced with those values of x for which the derivative f'[x] is equal to 0. This will return the list

{{{-1,17},0,-60},{{-I,5+12I},0,60I},{{I,5-12I} ,0,-60I},{{1,-7},0,60}.

We can immediately ignore the middle 2 sublists since they have complex numbers in them. From the first sublist we can read off at the point (-1,17) that the derivative is 0 and the second derivative is -60 - so (-1,17) is a local max. From the last list we see that at the point (1,-7), the derivative is 0 and the second derivative is 60 - so (-1,17) is a local minimum. All the algebra work of the problem done in a single command.

Mathematica can also handle mixed derivatives. If you want to take the derivative of some expression f with respect to x and then y, use the command D[f,x,y]. If you wanted to take the derivative with respect to x, then y, then y, then x again, use the command D[f,x,y,y,x]. Remember that mixed partial derivatives aren't always equal (although they almost always are in practice), so order could make a difference.

Recall that we can find local maxima, minima, and saddle points of a function f(x,y) by solving $f_x = 0$ and $f_y = 0$, and checking the resultant solutions (x,y) in the functions $\Delta = f_{xx} f_{yy} - f_{xy}^2$ and f_{xx}. We can do this for the function f(x,y)=$x^2 - y^3 + 6xy$ as follows: First define f[x_,y_]:=x^2-y ^3+6x y (you may want to use Clear[f] first if you've previously defined f). Then enter the command:

```
{ {x,y,f[x,y]}, D[f[x,y],x,x]D[f[x,y],y,y]-D[f[x,y],x,y]ˆ2, D[f[x,y],x,x]}
/. Solve[{D[f[x,y],x]==0],D[f[x,y],y]==0},{x,y}].
```

Let's analyze this (admittedly convoluted) expression. The list before the /. symbol is the list { (x,y,z),Δ, f_{xx} }, and then we are replacing x and y in this list with the set of solutions {x,y} from the system of equations $\{f_x = 0, f_y = 0\}$. The result of this evaluation is the list { {{0,0,0},-36,2},{{16,-6,-108},36,2}}. So at the point (0,0,0), $\Delta < 0$, so (0,0,0) is a saddle point. At the point (16,-6,-108), $\Delta = 36 > 0$ and $f_{xx} = 2 > 0$, so the point (18,-6,-108) is a local minimum for the function f(x,y). This may seem like a lot of work (that expression was pretty long), but to do another problem all you would have to do is redefine f[x,y] and then re-evaluate that one long cell.

Section Three - Integrals

Mathematica can handle a wide variety of integrals, both definite and indefinite.

Mathematica does not find indefinite integrals per se but it does find antiderivatives - the only difference is the arbitrary constant C. To find an antiderivative of f(x), enter the command Integrate[f[x],x]. You do not have to formally define a function f[x]; if you want to find an antiderivative of $3x^2$ simply enter Integrate[3xˆ2,x]. Or you can define f=3xˆ2 first, and then use Integrate[f,x]. Unfortunately, although multiple antiderivatives and mixed-variable antiderivatives will work just like they do in the D[ ] command, at each step Mathematica will assume the arbitrary constants are zero, which isn't really true for general antiderivatives. So if you are working a problem in which the constants of integration are important, you are better off using the DSolve[ ] command, which we will examine in Section Five.

Mathematica can handle virtually any antiderivative you throw at it. However, it may use functions you are not familiar with (it uses hyperbolic functions a lot where the techniques you learn in class may call for trigonometric substitution, for example). It may also apply identities like $\cot^{-1}$(x)=$\tan^{-1}$(1/x) to put things in what it believes is the simplest form. Remember, you can always check the answer to an antidifferentiation problem by taking the derivative of the result - and in Mathematica this is easy because you can follow the Integrate[ ] command with D[%,x] (the derivative of the last output). Mathematica will even integrate expressions with arbitrary constants in them, like $\int x^n dx$. In this case, Mathematica will return $\frac{x^{1+n}}{1+n}$ because it makes no assumptions about whether n=-1 or not (Integrate[1/x,x] will return Log[x] though).

Although Mathematica can perform virtually any integral you can find in a standard table, there are still integrals it cannot do in a "closed form" (as a finite sequence of basic functions) - simply because they cannot be done. Integrals of e^{-x^2} and $\frac{\sin(x)}{x}$ fall into this category. Mathematica may return these sorts of integrals in terms of special functions you can look up with the ? symbol (in these cases, the results involve Erf[x] and SinIntegral[x]), or it may just hand back the integral to you, essentially telling you that you are on your own.

Mathematica can also do definite integrals, whether they are proper or improper. To find the definite integral of f(x) from x=a to x=b (Infinity and -Infinity being possible values for b and a), enter Integrate[f[x],{x,a,b}]. For example, to find $\int_1^3 x^3 dx$, enter Integrate[xˆ3,{x,1,3}]. To integrate $\frac{1}{x^2}$ from x=2 to x=∞, enter Integrate[xˆ(-2),{x,2,Infinity}]. Mathematica will detect if an integral is improper and will tell you if it does not converge (for example, when entering Integrate[1/x,{x,-1,1}] to find $\int_{-1}^{1} \frac{1}{x} dx$).

Mathematica can do multiple definite integrals as well. To find $\int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x,y) dydx$ enter Integrate[f[x,y],{x,a,b},{y,c,d}]. Note than the order of the "limit lists" at the end are the same as the order of the integral signs in the original problem - written this way, the y-integral is done first and then the x-integral is done. This is important because written in this order, the y "limit list" can have limits defined in terms of x. For example, suppose you wanted to find the integral of $x^2 - 3y$ over the top half of the unit circle, which is bounded by y=0, y=$\sqrt{1-x^2}$,x=-1, and x=1. You could find this integral as Integrate[xˆ2-3y,{x,-1,1},{y,0,Sqrt[1-xˆ2]}]. Written mathematically, this would be $\int_{x=-1}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} (x^2 - 3y) dydx$. Integrals in three or more variables can be done the same way, as Integrate[f[x,y,z],{x,*xlimit1*,*xlimit2*},{y,*ylimit1*,*ylimit2*},{z,*zlimit1*,*zlimit2*}].

Section Four - Series

Mathematica can find many series for you. The notation for adding up the terms a[n] from n=a to n=b is Sum[a[n],{n,a,b}]. For example, if you wanted to add up the terms $\frac{1}{n^2}$ from n=1 to 1000, enter Sum[1/nˆ2,{n,1,1000}]. You can even use Infinity and -Infinity in limits, although Mathematica may tell you the series diverges. For example, you can try to add up the terms $\frac{1}{n^2}$ from n=1 to ∞ as Sum[1/nˆ2,{n,1,Infinity}] - you will get the exact sum $\frac{\pi^2}{6}$. If you try this with $\frac{1}{n^3}$, Mathematical will return Zeta[3] - the series converges (and if you follow the Sum with N[%] you'll get an approximation of the true sum), but there's no convenient known formula for the answer. If you try to do this with $\frac{1}{n}$, Mathematica will tell you the series diverges.

Mathematica can also find Taylor polynomials for you. To find the n-th order Taylor series for f(x) at x=a, enter Series[f[x],{x,a,n}]. To find the 4th order Taylor Series for Sin[x] at $\frac{\pi}{2}$, enter Series[Sin[x],{x,Pi/2,4}]. The result will look somewhat strange -

$$1-\frac{1}{2}(x-\frac{\pi}{2})^2+\frac{1}{24}(x-\frac{\pi}{2})^4+O[x-\frac{\pi}{2}]^5$$

The $O[x-\frac{\pi}{2}]^5$ term is present because Mathematica knows (from Taylor's Theorem) that the error in approximating Sin[x] with a fourth order polynomial at $\frac{\pi}{2}$ essentially behaves like $(x-\frac{\pi}{2})^5$ - that's what the "O" term is for. While it's good that the computer keeps track of that for you, you cannot really plug numbers into this. You can strip the "O" term from the polynomial using the command Normal[ ] - either when you enter the series (as Normal[Series[Sin[x],{x,Pi/2,4}]]) or by immediately following the Series[ ] output with Normal[%]. Either method will strip the "O" term and allow you to start plugging numbers in for x or plotting the Taylor polynomials.

Mathematica can even find Taylor polynomials in more than one variable. To find the Taylor polynomial for f(x,y) at the point (a,b) of order n in x and m in y, use the command Series[f[x,y],{x,a,n},{y,b,m}]. This type of series will also have a number of "O" terms in it, and these can again be stripped off with the Normal[ ] command.

Section Five - Differential Equations

Mathematica has a very powerful engine for handling differential equations in a single variable. The command DSolve[ ] is useful for solving differential equations (including "initial value problems") and for indefinite integration when you wish to keep track of arbitrary constants. Let *eqnlist* be a list of equations which relate the derivatives of y with respect to x; then to solve the differential equation for y in terms of x, use the command DSolve[*eqnlist*,y[x],x].

For example, suppose you wanted to solve the differential equation $y''+y=2x$ with the initial conditions y(0)=0 and y'(0)=1. The first thing you must do is to clear any previous definitions for y and x (you can do this in one shot with Clear[y,x]). To solve the equation, enter the command

```
DSolve[ {y"[x]+y[x]==2x,y[0]==0,y'[0]==1},y[x],x]
```

Notice that the equations (even the initial values) are given in the equation == format, and that y and its derivatives are written as if they were functions of x. The result of this particular command is the replacement rule { {y[x]->2x-Sin[x]}} - that is, the only solution to this problem is y=2x-sin(x).

If you want to find the indefinite integral of x*sin(x) with respect to x, you can use the command DSolve[{y’[x]==x Sin[x]},y[x],x]. The result of this command is the replacement rule {{y[x]->C[1]-x Cos[x]+Sin[x]}} - and it includes the arbitrary constant C[1]. If you wanted to find all functions whose second derivative was x*sin(x), you would use the command DSolve[{y”[x]==x Sin[x]},y[x],x]. The result of this command will have 2 arbitrary constants C[1] and C[2] in it.

To solve initial value problems, simply include the initial values as part of the equation list. If you have to find all functions y whose second derivative is $24x^2e^x$ and for which y(0)=1 and y’(0)=0, use the command DSolve[{y”[x]==24xˆ2 Exp[x],y[0]==0,y’[0]==1},y[x],x].

Chapter Six Exercises

1) Find $\lim_{x\to\pi} \frac{\sin(x)}{x}$, $\lim_{x\to 0} \frac{\sin(x)}{x}$, and $\lim_{x\to\infty} \frac{\sin(x)}{x}$.

2) Find $\lim_{x\to\infty} \frac{3x^2-3x+2}{6x^5-5x-1}$.

3) Find $\lim_{x\to\infty} \frac{2^x}{x^3+1}$.

4) Take the derivative of $x^3 - 6x^2 - 2x + 1$.

5) Take the second derivative of $x^2 e^x$.

6) Simplify the derivative of $x^{\frac{1}{\ln(x)}}$.

7) What is the fourth derivative of e^{x^2} at x=1?

8) Find the critical points of $x^4 - 4x^3 + 2x^2 - 9$.

9) Find all of the first and second order partial derivatives of $\frac{x^2y-y^2}{x^2+y^2}$.

10) Find the Taylor polynomials of tan(x) at 0 of orders 3, 5, and 10. Use these to estimate $\tan(\frac{1}{2})$.

11) Find an antiderivative of $x^2\sin(x)$.

12) Find an antiderivative of $\frac{1}{x^3-1}$.

13) Find an antiderivative of $\sum_{n=2}^{\infty} \frac{x^n}{n^2}$.

14) Find $\int_0^1 x^3e^x dx$.

15) Find $\int_1^\infty \frac{x^2}{e^x} dx$.

16) Numerically estimate $\sum_{n=2}^{\infty} \frac{n+1}{n^4}$.

17) Solve the differential equation $y'''' - y = 0$.

18) Solve the initial value problem $y'' + y = 0, y(0) = 3, y'(0) = 2$.

19) Solve the initial value problem $y'' - y = 0, y'(0) = 3, y'(0) = 2$.

20) Solve the differential equation $y''' + y = \sin(x), y(0) = 0, y'(0) = 1, y''(0) = 1$.

21) Define f to be a nickname for the solution to $y'' + y = 0, y(0) = 3, y'(0) = 2$ in a form you can manipulate (that is, take a derivative of and so on).

Chapter Seven - Linear Algebra

Mathematica has built-in functions for working in linear algebra. Matrices are represented in Mathematica by lists of lists and can be generated and displayed in Mathematica using the Table[ ] and MatrixForm[ ] commands as outlined in Chapter Three. If *mat* is a matrix, recall from Chapter Three that Dimensions[*mat*] will return the dimensions of *mat*.

Vectors in Mathematica are represented by lists of numbers. For example, {1,5,6} represents a vector in $\mathbf{R}^3$ and {1,5,3,2} represents a vector in $\mathbf{R}^4$. Vectors can be added and scaled as matrices (see below). The dot products of vectors v1 and v2 (which must have the same Length[ ]) is given by v1.v2. For example, {1,0,1}.{2,1,3}=1*2+0*1+1*3=5.

Matrix Operations

Let *mat1* and *mat2* be matrices.

Matrix Addition/Subtraction: If the dimensions of *mat1* and *mat2* are the same, then *mat1*+*mat2* will return a list which is the matrix sum of *mat1* and *mat2*. For example, {{1,2,3},{0,9,8}}+ {{3,3,3},{5,5,5}} will return the matrix {{4,5,6},{5,14,13}}. *mat1*-*mat2* will be the matrix difference of *mat1* and *mat2*.

Scalar Multiplication: If *mat1* is any matrix, then f**mat1* (or f *mat1*) is the matrix obtained by scaling *mat1* by f. This works is f is a real number, complex number, or even a function or variable.

Matrix Multiplication: If Dimensions[*mat1*]={m,n} (that is, *mat1* represents an m-by-n matrix) and Dimension[*mat2*]={n,k} (*mat2* is an n-by-k matrix), then *mat1.mat2* is the matrix product of *mat1* and *mat2*.

Transposing Matrices: Transpose[*mat1*] will return the transpose of *mat1* (*mat1* with its rows and columns interchanged).

Determinants: If *mat1* is a square matrix, then Det[*mat1*] is the determinant of *mat1*.

Minors of matrices: If *mat1* is any matrix and k is a positive integer, Minors[*mat1*,k] will return a list of the k-by-k minors of *mat1*; that is, a list of the determinants of all k-by-k submatrices of *mat1*.

Inverses: If *mat1* is a square matrix and if Det[*mat1*] is not 0, then Inverse[*mat1*] will be the matrix inverse of *mat1*. If you attempt to take the inverse of a singular matrix, Mathematica will return an error.

Identity Matrices: IdentityMatrix[n] generates the n-by-n identity matrix.

Finding Eigenvalues: If *mat1* is a square matrix, then Eigenvalues[*mat1*] will return a list of the eigenvalues of *mat1*, both real and complex. An alternate way of finding eigenvalues from the definition would be the command x/.Solve[Det[x*IdentityMatrix[Length[*mat1*]]-*mat1*]==0,x]. In practice you always use the Eigenvalues[ ] command, but this shows how you can combine other functions to do complex calculations.

Finding Eigenvectors: If *mat1* is a square matrix, the Eigenvalues[*mat1*] will return a list of the eigenvalues of *mat1* (remember, if *mat1* is an n-by-n matrix, this will be a list of at most n vectors). Eigenvectors may involve complex numbers in their components, as Mathematica always works over the complex numbers.

Solving Linear Equations: This is best done by the Solve[ ] command.

Chapter Seven Exercises

1) If u={1,0,3} and v={2,-1,3}, what is u.v?

2) If u={3,4,1,4} and v={-2,1,0,3}, what is u.v?

In the 3-8, let A= $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 2 & 1 \\ 0 & -6 & 5 \\ 1 & 2 & 3 \end{bmatrix}$.

3) Find A-B.

4) Find 2A+B.

5) Find AB-BA.

6) Find A^T.

7) Find det(A).

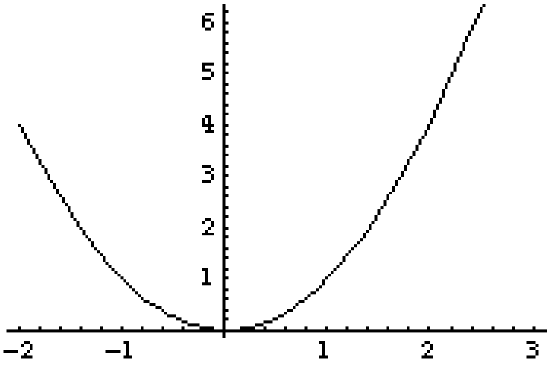
8) Find A^{-1}.

9) Let C= $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find the eigenvalues and eigenvectors of C.

Chapter Eight - Two-Dimensional Plots

Section One - Basic Plots

Elementary graphing in Mathematica is handled by the Plot[ ] function. If *expr* is an expression with x as its only variable, to create a graph of the equation y=*expr* as x goes from a to b, use the command Plot[*expr*,{x,a,b}]. To create a graph of the parabola y=x^2 as x goes from -2 to 3, you could enter Plot[xˆ2,{x,-2,3}]. Or you could formally define f[x_]:= xˆ2 first, and then use Plot[f[x],{x,-2,3}]. Or you could define f as the expression x^2 by using f=x^2 and then entering Plot[f,{x,-2,3}]. All of these commands should generate the graphic



Plot[ ] has a lot of options which will allow you to control exactly what the graph looks like. Using the basic Plot[ ] function allows Mathematica to use what it thinks is the best choice for these options.

You can create a plot of more than one function on the same set of axes by using lists. If you wished to plot the graphs of y=x, y=x^2, and y=sin(3x) on the same set of axes (say as x goes from -3 to 3), you would use the command Plot[{x,xˆ2,Sin[3x]},{x,-3,3}].

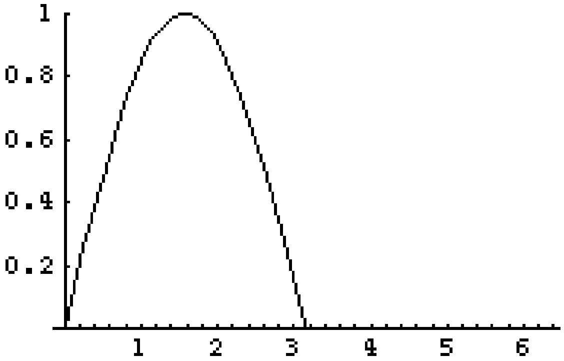
There's also no reason why x has to be the variable - Plot[tˆ2,{t,-2,3}] will generate exactly the same plot of a parabola as we did above.

Section Two - Plot Options

There are about 30 options for the Plot[ ] command which allow you to customize the way plots are generated. The options for Plot[ ] follow the following notation:
Plot[f[x],{x,a,b}, *option1->Value1,option2->Value2,...*]. You may enter or omit as many options as you want. A complete list of the options for the Plot[ ] command and their default values can be

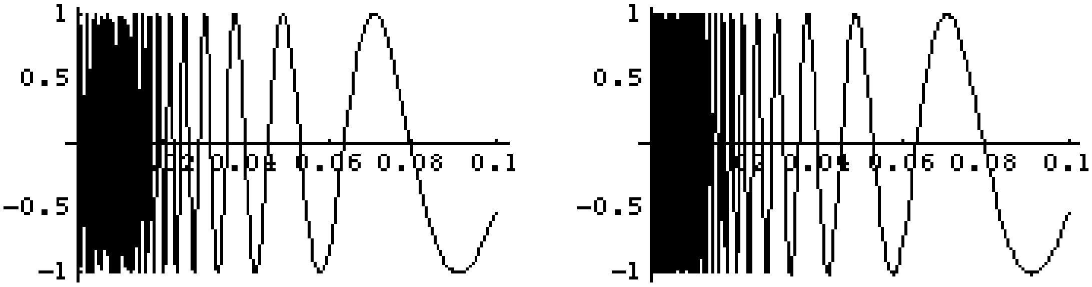
seen by evaluating the command Options[Plot]. Rather than describe every possible option, here is a list of how the most common ones are used.

Controlling the range of y-values in the graph: The option PlotRange controls which y-values will be shown. For example, if you want to show the y-values from 2 to 6, use the option PlotRange->{2,6}. This option is important for "zooming" in on certain features of the graph. For example, if a graph has a vertical asymptote (like tan(x) and 1/x do), Mathematica may try to plot very large ranges for y, say y going from -1000 to 1000. This scale will make it impossible to see any local details on the graph. The graph may have a local maximum at (2,3), but you'll never see it on that scale. Restricting the range of y-values will allow you to see more features of the graph. An example of this option would be Plot[Sin[x],{x,0,2Pi},PlotRange->{0,1}], which generates the graphic

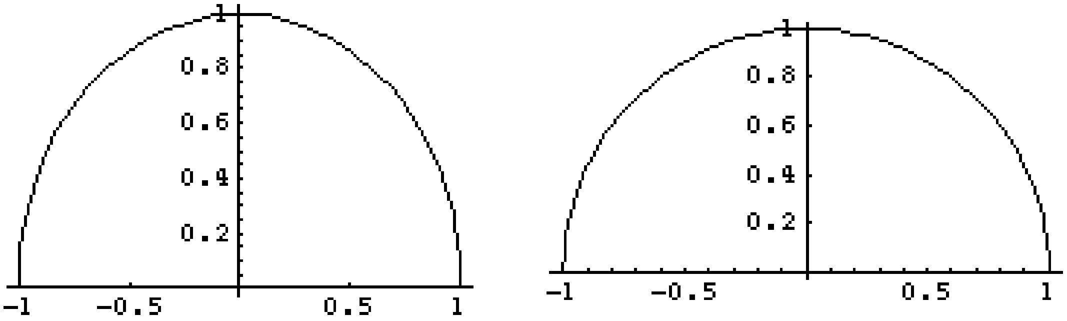


You can also force Mathematica to try to plot all y-values that occur in a graph (sometimes a graph might be "beheaded" by the edges of the plot) - to do this, use PlotRange->All.

Plotting more points: The option PlotPoints controls how many points are plotted in the graph. Mathematica creates graph by plotting a lot of points and then trying to connect them cleverly. The number of points it uses is PlotPoints. The default value is 25. If a graph has very rapid changes in it (like y=sin(1/x) does near x=0) or the range of x-values is large, 25 points may not be enough to capture the true behavior of the graph. Using more points by using the option PlotPoints->50 or PlotPoints->100 may make the picture a bit better. An example would be Plot[Sin[1/x],{x,-.2,.2},PlotPoints->100]. Of course, the more points you plot the longer it takes to graph and the more memory Mathematica may need. Below are two graphs - one of sin(1/x) with no PlotPoints specified (on the left) and one with PlotPoints->100 (on the right). Notice how much more even the graph on the right is - even though the curve goes wild as x gets close to 0 in both pictures, the oscillations seem much more even in the second graph.

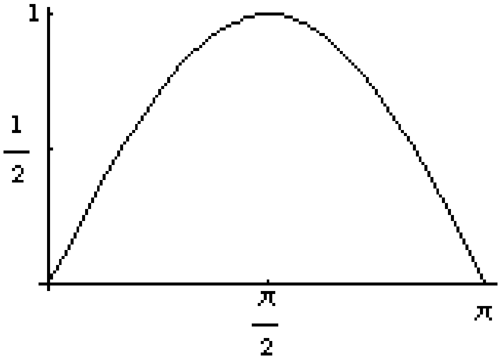


Controlling the relative height and width of the graph: AspectRatio is the ratio of height to width of the graph. Mathematica creates plots which are roughly 3/5 as tall as they are wide, primarily for aesthetic reasons. To do this, it may use a different scale on the x-axis than the y-axis. This will distort the graph somewhat - circles may appear as ellipses, for example. Setting AspectRatio->Automatic will create plots with the same scale on both axes, preserving the shapes of graphs. Setting AspectRatio->1 will create a plot as tall as it is wide. The following graphic is two shots of the top half of the unit circle - on the left using the default value of AspectRatio, on the right with AspectRatio->Automatic.

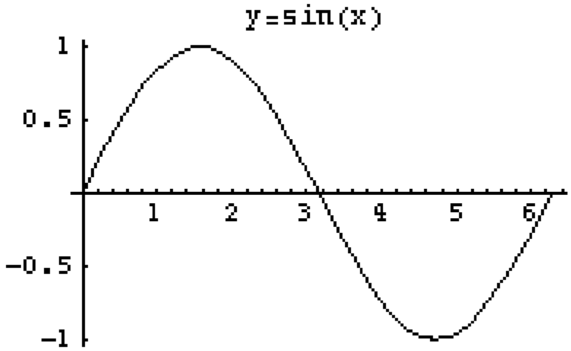


Removing Axes: If you do not want axes in your graph (if they obscure an important part of the graph, perhaps), you can remove them with the Axes option by setting Axes->None.

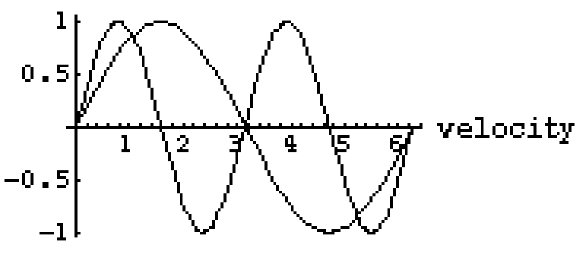
Tick Marks on Axes: The tick marks on the axes are controlled by the option Ticks. Setting Ticks->None will remove all tick marks on the axes. If *list1* is a list of what x-ticks you want and *list2* is a list of the y-ticks you want, setting Ticks->{*list1*,*list2*} will force Mathematica to use these tick marks and no others. The Table[ ] and Range[ ] commands are useful for generating the lists for the tick marks. Some examples are Plot[Sin[x],{x,0,Pi},Ticks->{{0,Pi/2,Pi},{0,1/2,1}}] and Plot[x^2,{x,-3,3},Ticks->{Table[i,{i,-3,3}], Table[j,{j,0,9}]}]. The plot of the sine curve in this example is



Labeling Plots: The option PlotLabel allows you to put a label on plots. The label you use should have double quotes around it; PlotLabel->“y=sin(x)”, for example. A full example is Plot[Sin[x],{x,0,2Pi},PlotLabel->“y=sin(x)”].



Labeling Axes: AxesLabel places labels on the axes. The option AxesLabel->{“*xlabel*”,“*ylabel*”} labels the x-axis with *xlabel* and the y-axis with *y-label*. An example is Plot[{Sin[x],Sin[2x]},{x,0,2Pi}, AxesLabel->{“velocity”,“time”}]. Just as for PlotLabel, the labels must be surrounded by double-quote marks.



You can combine these options in a single graph - for example, you can use the command Plot[Sin[x],{x,0,2Pi},PlotRange->{-3/2,3/2},AxesLabel->{“x”,“y”}, PlotLabel->“y=sin(x)”, AspectRatio->Automatic, Ticks->{Table[i Pi/4,{i,0,8}],{-3/2,-1,-1/2,0,1/2,1,3/2}}].

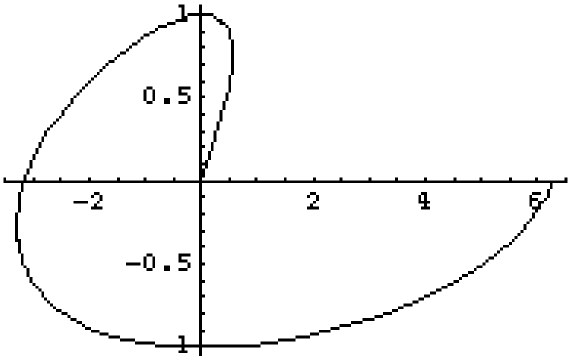
You may not need to know to use an option like AspectRatio until after you've done an initial plot. You can always go back and edit and then reevaluate the cell, of course. You can also use the Show[ ] command which allows you to replot with different options. For example, suppose you wanted to plot the top half of a circle of radius two with the command Plot[Sqrt[4-xˆ2],{x,-2,2}]. The plot will look somewhat elliptical because Mathematica is using different scales in the x- and y-directions. You can immediately replot by using the command Show[%,AspectRatio->Automatic] as the next command after the original plot. You can alter as many of these options at once as you want (except for PlotPoints) by using Show[ ].

You can assign nicknames to plots just like you can numbers and functions. You could enter f=Plot[Sqrt[4-xˆ2],{x,-2,2}] and then use Show[f,AspectRatio->Automatic], for example.

Show[ ] can be used to plot multiple curves on the same set of axes as well. If you let f=Plot[Sqrt[4-xˆ2],{x,-2,2}] and g=Plot[Sin[x],{x,-2,2}], then Show[f,g] will combine the plots into a single graph.

Section Three - Parametric Plots

Mathematica can generate plots parametrically using the command ParametricPlot[ ]. If the x-coordinate of a point is given by f(t) and the y-coordinate is given by g(t), then the parametric plot from t=a to t=b is generated by ParametricPlot[{f[t],g[t]},{t,a,b}]. For example, to plot the curve defined by x=t*cos(t) and y=sin(t) as t goes from t=0 to 2π, use the command ParametricPlot[{t Cos[t],Sin[t]},{t,0,2Pi}]. This parametric plot should look like this:



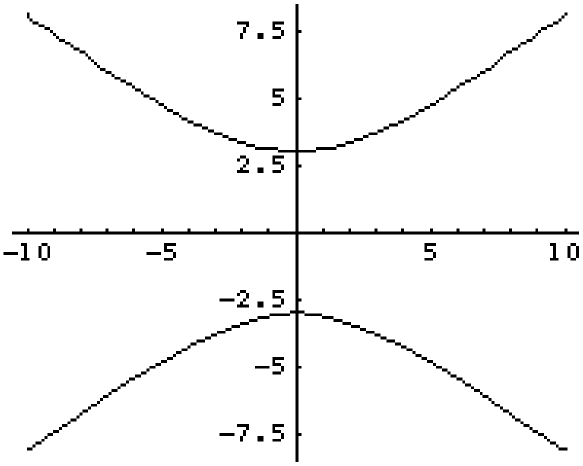
The options for ParametricPlot[ ] are the same as for Plot[ ] and entered in exactly the same fashion (including the use of Show[ ] to change options or combine plots). For example, you could enter the command:

ParametricPlot[{t Cos[t],Sin[t]},{t,0,2Pi}, AspectRatio->Automatic, AxesLabel->{“x”,“y”}].

One common type of parametric plot is the the graph of a curve in polar coordinates. Such curves can be graphed using ParametricPlot[] and the formulas for translating x and y in terms of r and θ or by using a special command PolarPlot[] which can be loaded in from the Graphics`Graphics`package.

Section Four - Plotting the Graphs of Equations

The unit circle is given by $x^2 + y^2 = 1$. You cannot plot this as the graph of a single function - the best you can do is to solve for y as $\pm\sqrt{1-x^2}$ and then use Plot[{Sqrt[1-xˆ2],-Sqrt[1-xˆ2]}, {x,-1,1}]. You can plot the graph of the equation $x^2 + y^2 = 1$ directly using the command ImplicitPlot[]. Unlike Plot[], ImplicitPlot[] resides in one of the add-on packages, so you must first load the package by evaluating <<Graphics`ImplicitPlot`. Once you have done this, you can then use the ImplicitPlot[] command as follows. Let *eqn* by an equation like xˆ2+yˆ2==1. To plot the graph of the equation as x goes from a to b, use the command ImplicitPlot[*eqn*,{x,a,b}]. For example, to plot the unit circle above you would enter ImplicitPlot[xˆ2+yˆ2==1,{x,-1,1}] (you could plot it from x=-2 to 2, but this would just include some empty space around the circle). To plot the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$ as x goes from -10 to 10, you would use the command ImplicitPlot[yˆ2/9-xˆ2/16==1,{x,-10,10}]. This hyperbola should look like:



ImplicitPlot[] uses the same options as Plot[] does (AspectRatio, AxesLabel, and so on).

Chapter Eight Exercises

1) Graph $y = x^3$ as x goes from -2 to 3.

2) Graph y=sin(x) and cos(2x) on the same set of axes as x goes from 0 to 10π.

3) Graph y=cos(x) from 0 to 2π. Use tick marks $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ on the x-axis and $0, \frac{1}{2}, 1$ on the y-axis. Label the plot "y=cos(x)".

4) Plot y=$\frac{1}{x^3-x}$ from x=-5 to 5. Restrict the range of y-values displayed to -5 to 5.

5) Create a plot of y=$\sqrt{9-x^2}$ as x goes from -3 to 3. Use AspectRatio->Automatic and label both axes.

6) Create a plot of the parametric curve defined by x=t cos(t), y=t sin(t) as t goes from 0 to 10. Use AspectRatio->Automatic, and have Mathematica sample the functions 50 times to create the plot.

7) Create a plot of the curve defined by x=sin(2t), y=sin(t) as t goes from 0 to 20.

8) Create a graph of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ as x goes from -4 to 4. Preserve the shape of the graph.

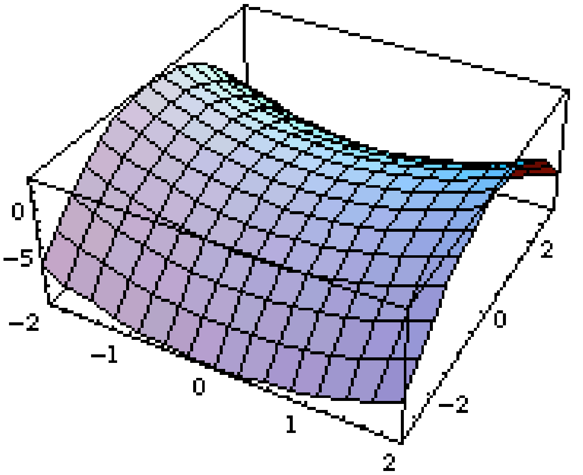
9) Create an plot of the curve $y^2 = x^3 - 16x$ as x goes from -20 to 20. Label the axes of the plot.

10) Plot the solution to the initial value problem $y' = 2y - y^2, y(0) = \frac{1}{2}$ as x goes from 0 to 5, showing y-values from 0 to 3.

Chapter Nine - Three-Dimensional Plots

Section One - Basic 3-D Plots

The command for creating three-dimensional plots is Plot3D[ ]. To create a plot of the surface z=f(x,y) over the rectangle consisting of x-values from x=a to x=b and y-values from y=a to y=b, use the command Plot3D[f[x,y],{x,a,b},{y,a,b}]. For example, to create a plot of the hyperbolic paraboloid z=$x^2 - y^2$ as x goes from -2 to 2 and y goes from -3 to 3, you would enter the command Plot3D[xˆ2-yˆ2,{x,-2,2},{y,-3,3}]. You should see the following graphic:



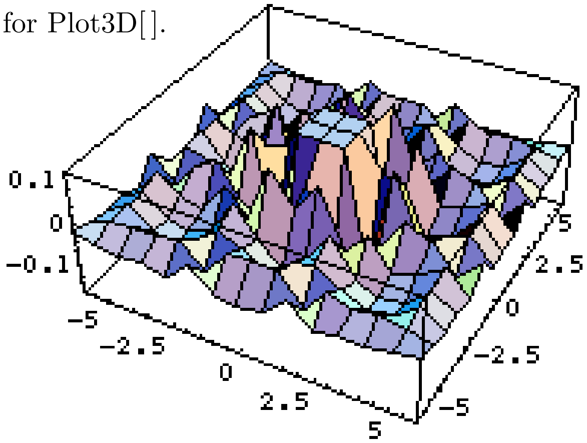
The format for Plot3D[ ] is exactly the same as for the Plot[ ] command, except you have to add the second list for the variable y and its range. Another way to plot this function would to first define f=xˆ2-yˆ2, and then enter Plot3D[f,{x,-2,2},{y,-3,3}]. This approach would be useful if you are going to work with $x^2 - y^2$ - taking derivatives, plugging in numbers with the /. notation, and so on.

Unlike the Plot[ ] command, you cannot use Plot3D[ ] to plot more than one surface in the same 3-dimensional box. You can, however, combine 3-dimensional plots using the Show[ ] command, just as you can for plots.

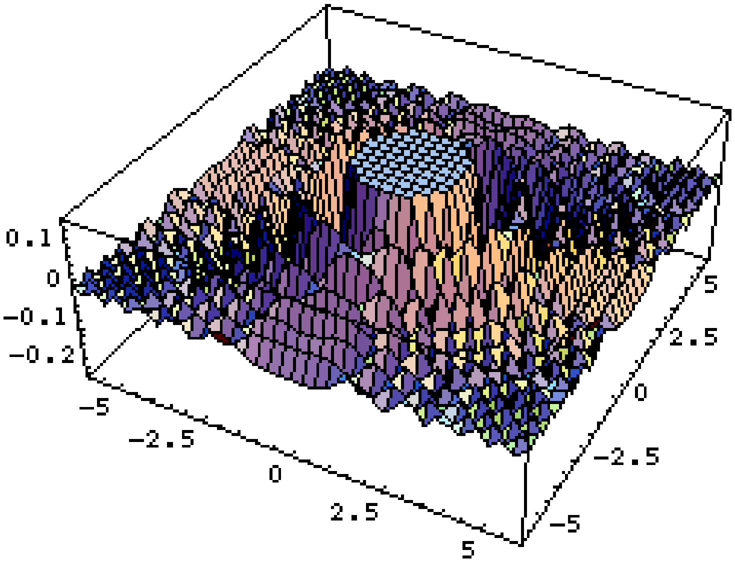
For example, define a= Plot3D[xˆ2-yˆ2,{x,-2,2},{y,-3,3}] (the paraboloid above) and define b=Plot3D[-xˆ2,{x,-3,3},{y,-3,3}] (the graph of this is a cylindrical surface generated by a parabola which opens downward). If you then evaluate Show[a,b], you will create a 3-dimensional plot of the two surfaces in the same space. You could achieve the same effect using the % notation - just follow the two 3-D plot commands with Show[%,%%].

Section Two - Plot3D[] options

In the following, define f=Sin[x^2+y^2]/(x^2+y^2). Evaluate Plot3D[f,{x,-6,6},{y,-6,6}] - you should see the graphic below. There are several flaws in this graphic which will illustrate some of the important options for Plot3D[].

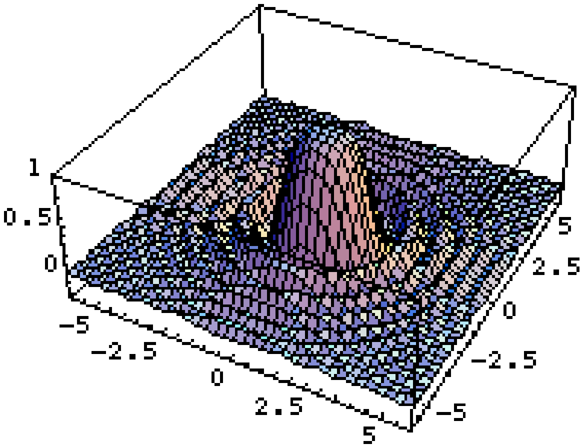


The plot is jagged: The plot is very jagged, filled with sharp edges and corners. This is because like it does in Plot[], Mathematica creates 3-D plots by plotting a bunch of points and then connecting the values (this time with polygons rather than line segments). The function $\frac{\sin(x^2+y^2)}{x^2+y^2}$ changes very rapidly away from the origin, so Mathematica is not using enough points to get a good idea of what the surface looks like. We can increase the number of points Mathematica samples using the option PlotPoints, just as for Plot[]. If you enter Plot3D[f,{x,-6,6},{y,-6,6}, PlotPoints->40], this will replot the surface much more smoothly, and you will be able to see the "ripples" in the surface around the origin. This plot is given below.



When increasing PlotPoints, keep in mind than the work it takes to create the surfaces goes up as the -square- of the number of points being sampled. If you go from PlotPoints->20 to PlotPoints->40, it will take roughly four times as much time and memory to create the plot.

Not all z-values are shown: Even if you increase the number of points being plotted (as in the plot above), you will notice that the part of the surface over the origin is "beheaded" - it is not shown and the result looks completely flat. Mathematica has its own rules for what it considers "optimal" ranges of z-values to plot. You can override this with the PlotRange option. PlotRange->{*zmin*,*zmax*} will tell Mathematica to cut the surface off outside of the z-range *zmin* -*zmax*, and PlotRange->All will force it to try to plot the range of all z-values in the surface. If you evaluate Plot3D[f,{x,-6,6},{y,-6,6},PlotPoints->40,PlotRange->All], you will see why Mathematica cut the original surface off. This plot is:

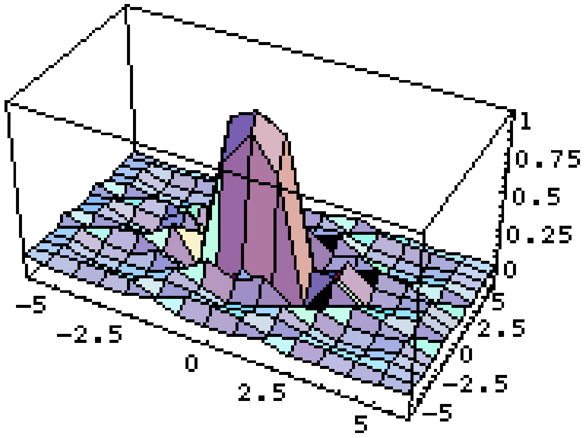


There is one large spike over the origin, which dwarfs the smaller ripples in the surface to the point where they are not easy to make out. A similar effect can be created by viewing the z-values from -1 to 1 with the command Plot3D[f,{x,-6,6},{y,-6,6},PlotPoints->40,PlotRange->{-1,1}].

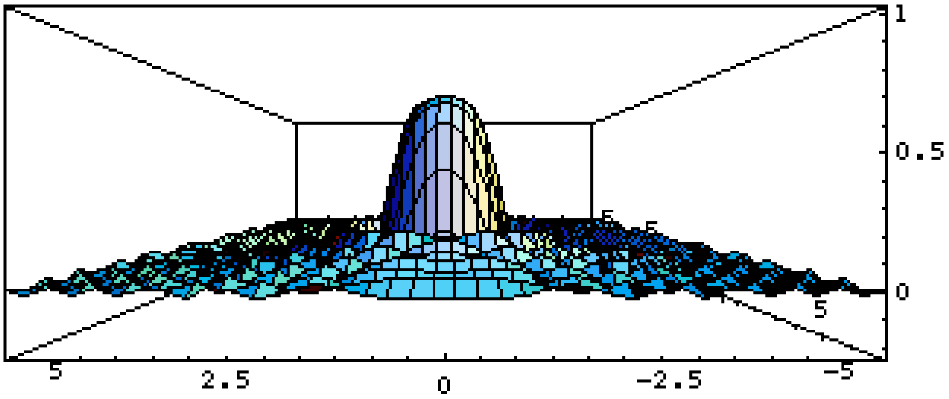
The bounding box: In all of the Plot3D[ ] commands above, Mathematica draws a box around the plot. You can remove this box with the option Boxed->False. This will not remove the axes on the edges, just the other edges of the box. For an example, enter the command Plot3D[f,{x,-6,6},{y,-6,6},PlotPoints->40,Boxed->False].

The axes: Axes are drawn in the edges of the bounding box so they are not hidden behind the surface. There are two options which deal with the axes themselves. If you wish to remove the axes (that is, not have the axes distinguished from the other edges of the bounding box), use the option Axes->False. Unless you combine this with the Boxed->False option, the lines will still be drawn without tick marks. To add labels to the axes, use the command AxesLabel->{*xlabel ,ylabel ,zlabel*}. Just as for Plot[ ], the labels you use should be enclosed in double-quote marks. For example, to remove the bounding box and tell which axis is which in the previous plot evaluate Plot3D[f,{x,-6,6},{y,-6,6},PlotPoints->40,PlotRange->All, Boxed->False, AxesLabel->{"x-axis","y-axis","z-axis"}]. The graphic this produces may be very compressed - to make it larger, first click on the graphic and then left-click on the lower right corner and drag to change the size.

Changing the relative dimensions of the plot: Mathematica tries to preserve the scales on the x-axis ,y-axis, and z-axis when possible. But you may want to override its decisions. For example, you may want a plot which is 3 times as tall as normal, or one where the y-direction is twice as long as the x-direction. You can do this with the option BoxRatios. BoxRatios->{a,b,c} creates a plot whose relative dimensions are a in the x-direction, b in the y-direction, and c in the z-direction. Using BoxRatios->{1,1,3} creates a plot whose length in the z-direction is 3 times that of the x- and y- directions. Using BoxRatios->{1,2,3} creates a plot whose length in the y-direction is twice that in the x-direction and whose length in the z-direction is three times than of the length in the x-direction. BoxRatios is literally a set of ratios - using BoxRatios->{1,2,1} will yield exactly the same results as using BoxRatios->{2,4,2} will, since the ratios x-to-z, y-to-z, and x-to-z are the same in each option. The following picture is the result of evaluating the command Plot3D[f,{x,-6,6},{y,-6,6},PlotRange->All,BoxRatios ->{2,1,1}]. Notice how the scales at the base of the graph have been distorted - the x-axis is now twice as large as the y-axis.



The viewpoint you look at the surface from: In all of the 3-D plots generated so far, the vantage you view the surface from is the same - you are viewing the surface from a point which is above the fourth quadrant of the xy-plane (this is much easier to figure out if you label the axes using AxesLabel). You may want to view the surface from other points - from directly below or above, for example. You can control the vantage point using the option ViewPoint. If you use the option ViewPoint->{a,b,c}, you will view the surface from a point on the ray starting at the origin (0,0,0) through the point (a,b,c). For example, to view the surface from up on the z-axis, set ViewPoint->{0,0,1}. To view it from underneath, set ViewPoint->{0,0,-1}. If you wish to view the surface from above the 1st quadrant in the xy-plane, try ViewPoint->{1,1,1}. The following graphic is generated using ViewPoint->{0,1,0} - we're looking at the surface from somewhere on the y-axis.



Mathematica has another way to select a viewpoint. Under the **Input** menu, there is a selection called 3D Viewpoint Selector. If you select this, a new window will pop up with a cube with labeled sides. Left-click somewhere on the cube and drag it - this will rotate the cube, giving you a hands-on way of visualizing viewpoints (this is easier to reconcile with coordinates if the dialog button "Cartesian" is selected, so it will show you coordinates in the usual way). Once you've rotated the cube so you see it from a vantage point you think will be useful, click the "Paste" button. This will insert the appropriate ViewPoint->{whatever} directly into your document at the point where the cursor was - the only thing it won't do for you is put in the appropriate comma. If you wanted to use the viewpoint selector for the surface we've defined by f, you would enter the partial command Plot3D[f,{x,-6,6},{y,-6,6},|], leaving the cursor where the | is. (don't actually type the |). Go into the viewpoint selector, drag the cube, and hit "Paste". The cell is now ready to evaluate and when you evaluate it you will get the view you wanted.

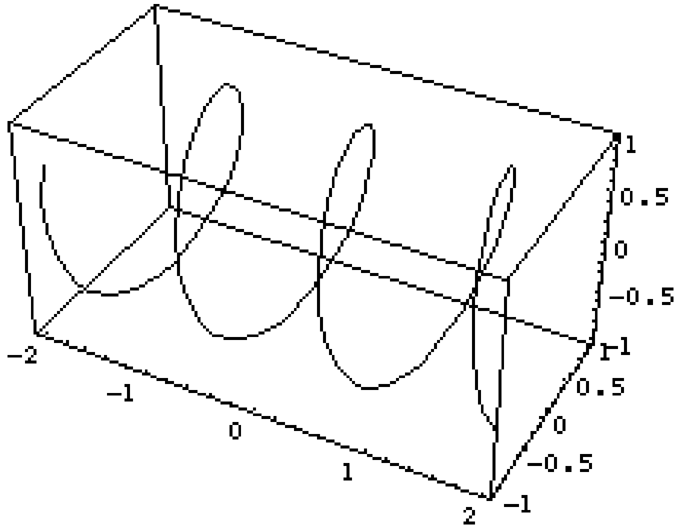
There are many other options for Plot3D[] which control other aspects of the plot like how the surfaces are colored and how lights are placed around the surface to produce shadows. You can get a complete listing of these options and their default values by evaluating the command Options[Plot3D].

The Show[] command can be used to alter options in a 3-D plot the same was as for the Plot[] command. For example, you could first evaluate Plot3D[f,{x,-6,6},{y,-6,6}], then Show[%, PlotRange->All], then Show[%,Boxed->False,AxesLabel->{"x-axis","y-axis", "z-axis"}].

Section Three - Parametric 3D Plots

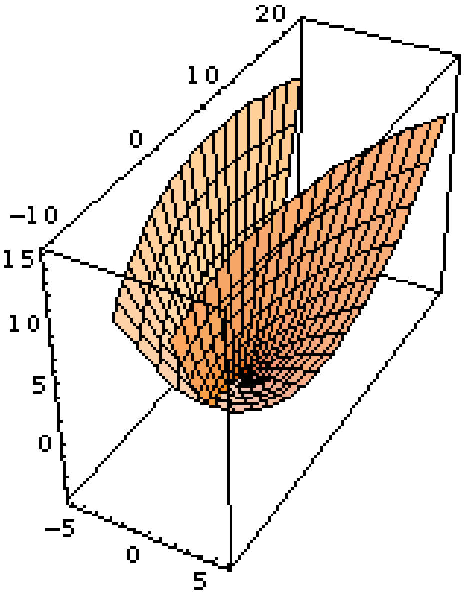
Mathematica can also do parametric plot in 3-space. Unlike parametric plots in the plane, there are two different types of parametric plots you can do. You can parametrically plot a curve (where the x-, y-, and z-coordinates all depend on a single variable, say *t*), or a surface (where the x-, y-, and z-coordinates of points are functions of a pair of variables, say *u* and *v*). Both types of parametric plots are done with the same command - ParametricPlot3D[].

Plotting Parameterized Curves: To a plot a curve whose coordinate functions are x=f(t), y=g(t), and z=h(t) over the t-interval *tmin-tmax* , use the command ParametricPlot3D[{f[t],g[t],h[t]},{t,*tmin* , *tmax* }]. For example, to create a plot of the helix defined by x=y/5, y=cos(t), and z=sin(t) as t goes from -10 to 10, you would use the following command: ParametricPlot3D[{t/5,Cos[t],Sin[t]},{t,-10,10}]. This curve is shown below:



There is nothing special about using t for the variable in a curve defined using parameters - it's just a common usage.

Plotting Parametric Surfaces: Plotting surfaces is very similar to plotting parameterized curves. To plot the surface whose coordinates are given by x=u+v, y=u^2-uv, z=v^2-u^2 for values of u between -2 and to and values of v from -4 to 4, you would use the command ParametricPlot3D[{u+v,u^2-u v, v^2-u^2},{u,-2,2},{v,-4,4}]. This particular surface looks like:



Again, there nothing special about the use of the u and v as the independent variables - it's just a common convention.

Plotting surfaces defined by parameterizations takes a lot of horsepower, so you may have to wait a few moments for the entire plot to be generated. If this is the case, you will see the phrase "Rendering" appear in the title bar of the Mathematica window.

ParametricPlot3D[] uses many of the same options as Plot3D[] - Boxed, Axes, AxesLabel, BoxRatios, and especially PlotPoints and ViewPoint. Surfaces defined using parameters often change their height very rapidly, so it is fairly common to use the PlotPoints option to get a smoother picture of the surface (remember, as you increase the value of PlotPoints, it takes more time for the computer to get the plot). These types of surfaces and curves often wrap around themselves and sometimes intersect themselves, so it is usually a good idea to construct several different plots with different ViewPoint values so you can see underneath and around the surfaces/curves - and you can used Axes->False and Boxed->False to remove axes and bounding boxes of they get in the way.

Chapter Nine Exercises

1) Plot $z = x^3 - y^3$ as x and y go from -3 to 3.

2) Replot the function from problem 1 but restrict the z-values displayed to the range -4 to 6.

3) Create a plot of $z = x^2 - y^2$ as x and y go from -3 to 3 with no bounding box whose scale in the z direction is 5 times larger than that of the x- and y- directions.

4) Create a plot of $z = \sin(e^{xy})$ as x and y go from -2 to 2. Make Mathematica sample enough points to get a relatively smooth picture of the surface, and label the axes of the plot.

5) Create a plot of the curve defined by x=sin(t), y=cos(t), z=sin($\sqrt{2}$t) as t goes from 0 to 10π. Label the axes and make Mathematica sample enough points to get a smooth curve.

6) Create a plot of the surface defined by $x = \frac{\sin(u)}{\sqrt{v}}, y = \cos(2u), z = v$ as u goes from 0 to 2π and v goes from 1 to 9.

Solutions to Exercises

The answers to the exercises are given as a Mathematica command or sequence of commands. If a problem takes more than one command, “followed by” or “and” indicate the sequence of commands. If a problem involves defining a function or nickname, it would probably be a good idea to use Clear[ ] to remove the definitions before going on to the next exercise.

Chapter One Exercises

1) N[Pi^E] followed by N[E^Pi] and see which is bigger.

2) Evaluate I^2, I^3, I^4, I^5, and I^6.

3) N[E,15]

4) <<Graphics`ImplicitPlot`followed by <<Graphics`Polyhedra`

5) <<Algebra`ThisIsNotARealPackage`   This will give you an error message.

Chapter Two Exercises

1) (256^2-8128)/40

2) Sqrt[964]

3) Abs[(2^4-3^3)/(6^7-7^6)]

4) Evaluate 20!

5) Binomial[20,4]

6) N[Sin[1]] followed by N[Sin[90]]. The latter is not 1 because the 90 is 90 radians, not degrees.

7) Cos[I Log[2]]

8) ArcCos[1/2] followed by N[ArcSin[3/5]]

9) Tan[ArcSin[1/7]] followed by N[%]

10) N[Log[3]] followed by N[Log[10,3]]

11) N[Sinh[1]] followed by N[Cosh[1]]

12) f[x_]:=Sin[x]/(Cos[x]+3)

13) g[x_]:= x^(1/Log[x]) (g[2] and g[3] will both return E).

14) h[x_,y_]:=x^2-sin(x y) followed by h[0,3]

15) ?f followed by ?g followed by ?h

16) Clear[f] followed by Clear[g] followed by Clear[h]

17) The k[x_] expression defines a function you can evaluate at values like k[3] or k[t+1], but k=3x-4 defines k as a nickname for 3x-4.

18) x=3 defines x as a nickname for 3, but x==3 is a True/False test for whether the variable x is equal to 3.

19) MyCubeRoot[x_]:= Sign[x] Abs[x]^(1/3)

Chapter Three Exercises

1) Range[30] or Table[i,{i,1,30}]

2) Table[2i+1,{i,0,29}]

3) a={1,2,-3,x,5,y} followed by a[[3]] and a[[6]]

4) Table[2^i,{-10,20,5}]

5) {{1,1/2,1/3,1/4,1/5},{2,1,2/3,1/2,2/5},{3,3/2,1,3/4, 3/5},{4,2,4/3,1,4/5}}

6) a=Table[i/j,{i,1,4},{j,1,5}] followed by Length[a] and Dimensions[a]

7) Union[{1,2,3,1,2}]

8) Intersection[Table[4i,{i,1,100}],Table[5j,{j,1,75}]]

9) a={{1,3,1},{2,4,1},{3,6,2}} followed by MatrixForm[a]

10) f=3x^5-15x+5 followed by f/. x->1, f/.x->2, and f/.x->3

11) Sin[x y]-x^2 y /. {x->Pi/4,y->2}

12) x^4-x^3 /. Table[{x->i},{i,-1,5}]

13) x^2+2x y-y^2-6x+4y+1 /. {x->x1+h, y->y1+k}

Chapter Four Exercises

1) Simplify[((x+h)^4-x^4)/h] followed by %/.{h->0}

2) f=Expand[x^2+2x y-y^2-6x+4y+1 /. {x->x1+h, y->y1+k}] followed by Coefficient[f, x1 y1] followed by Coefficient[f,x1] and Coefficient[f,y1]. Notice that the x1 coefficient involves y1 and vice versa. The coefficient of just the first degree term x1 can be obtained by Coefficient[f,x1]/.{y1->0} and the coefficient of the first degree term y1 can be obtained by Coefficient[f,y1]/.{x1->0}

3) Apart[x^12/(x^8-1)] followed by Together[%]

4) Factor[x^20-1]

5) FactorInteger[2^126-1]

6) PolynomialDivision[x^6-x^4+7,x^3-1,x]

7) Cancel[(x^12-1)/(x^9-1)]

8) Simplify[(x+2)^3-(x-1)^2+(x+9)^4-1]

Chapter Five Exercises

1) Solve[{3x+2y==1,6x-y==10},{x,y}]

2) Solve[{3x+2y==1,6x+4y==5},{x,y}]

3) Solve[{3x+2y==1, 9x+6y==3},{x,y}]

4) Solve[{3x+3y-z==1,x-3y+2z==0,x-4y-z==-3},{x,y,z}]

5) Solve[{x^2+y^2==4,y==3x+1},{x,y}]

6) Solve[x^2+2x-6==0,x]

7) Solve[x^2+4x+10==0,x]

8) Solve[x^3-4x+1==0,x]

9) Solve[x^4+3x^2-6x+2==0,x]

10) NSolve[x^4+3x^2-6x+2==0,x]

11) NSolve[3x^5-15x-5==0,x]

12) Solve[x==10Log[x],x] followed by N[%]

13) x^2-1 /. Solve[x^3-x^2-4x+4==0,x]

Chapter Six Exercises

1) Limit[Sin[x]/x,x->Pi], Limit[Sin[x]/x,x->0], and Limit[Sin[x]/x,x->Infinity]

2) Limit[(3x^2-3x+2)/(6x^5-5x-1),x->Infinity]

3) Limit[2^x/(x^3+1),x->Infinity]

4) D[x^3-6x^2-2x+1,x]

5) D[x^2 Exp[x],x,x] or D[x^2 Exp[x],{x,2}]

6) Simplify[D[x^(1/Log[x]),x]]

7) D[Exp[x^2],{x,4}] /. x->1

8) x/. Solve[D[x^4-4x^3+2x^2-9,x]==0,x]

9) f=(x^2 y-y^2)/(x^2+y^2) followed by D[f,x], D[f,y], D[f,x,x], D[f,y,y], and D[f,x,y].

10) Normal[Series[Tan[x],{x,0,3}]] followed by Normal[Series[Tan[x],{x,0,5}]] followed by Normal[Series[Tan[x],{x,0,10}]] followed by %%% /. x->1/2 followed by %%% /. x->1/2 followed by %%% /. x->1/2

11) Integrate[x^2 Sin[x],x]

12) Integrate[1/(x^3-1),x]

13) Integrate[Sum[x^n/n^2,{n,2,Infinity}],x]

14) Integrate[x^3 Exp[x],{x,0,1}]

15) Integrate[x^2/Exp[x],{x,1,Infinity}]

16) N[Sum[(n+1)/n^4,{n,2,Infinity}]]

17) DSolve[y''''[x]-y[x]==0,y[x],x]

18) DSolve[{y''[x]+y[x]==0,y[0]==3,y’[0]==2},y[x],x]

19) DSolve[{y''[x]-y[x]==0,y’[0]==3,y’[0]==2},y[x],x]

20) DSolve[{y'''[x]+y[x]==Sin[x],y[0]==0,y’[0]==1,y''[0]==1},y[x],x]

21) f= (y[x]/.DSolve[{y''[x]+y[x]==0,y[0]==3,y’[0]==2},y[x],x])[[1]]

The y[x]/. command tells Mathematica to replace y[x] with the solution from DSolve[ ] - but this is a list, so the [[1]] at the end returns the first (and only) element of the list.

Chapter Seven Exercises

1) {1,0,3}.{2,-1,3}

2) {3,4,1,4}.{-2,1,0,3}

For 3-8, define a={{1,2,3},{4,5,6},{2,1,5}} and b={{3,2,1},{0,-6,5},{1,2,3}}. To view the results as matrices, put the commands inside a MatrixForm[ ] command.

3) a-b

4) 2a+b

5) a.b-b.a

6) Transpose[a]

7) Det[a]

8) Inverse[a]

9) Define c={{1,2,1},{2,6,1},{1,1,0}}, then evaluate Eigenvalues[c] and Eigenvectors[c]

Chapter Eight Exercises

1) Plot[x^3,{x,-2,3}]

2) Plot[{Sin[x],Cos[2x]},{x,0,10Pi}]

3) Plot[Cos[x],{x,0,2Pi},Ticks->{{0,Pi/2,Pi,3Pi/2,2Pi},{0,1/2,1} },PlotLabel->“y=cos(x)”]

4) Plot[1/(x^3-x),{x,-5,5},PlotRange->{-5,5}]

5) Plot[Sqrt[9-x^2],{x,-3,3},AspectRatio->Automatic, AxesLabel->{“x-axis”,“y-axis”}]

6) ParametricPlot[{t Cos[t], t Sin[t]},{t,0,10},AspectRatio->Automatic, PlotPoints->50]

7) ParametricPlot[{Sin[2t],Sin[t]},{t,0,20}]

8) After loading in the ImplicitPlot package (see solutions to Chapter One Exercises), use ImplicitPlot[x^2/9+y^2/4==1,{x,-4,4}]. Or you could solve for y and use Plot[{2Sqrt[1-x^2/9],-2Sqrt[1-x^2/9]},{x,-4,4}] (this will cause Mathematica to give some errors, but it should still plot).

9) Load the ImplicitPlot Package and use ImplicitPlot[y^2==x^3-16x,{x,-20,20}, AxesLabel->{ “x-axis”,“y-axis”}]

10) Define f by f=(y[x]/.DSolve[{y’[x]==2y[x]-y[x]^2,y[0]==1/2},y[x],x])[[1]], and then use Plot[f,{x,0,5},PlotRange->{0,3}]

Chapter Nine Exercises

1) Plot3D[x^3-y^3,{x,-3,3},{y,-3,3}]

2) Show[%,PlotRange->{-4,6}]

3) Plot3D[x^2-y^2,{x,-3,3},{y,-3,3},Boxed->False, BoxRatios->{1,1,5}]

4) Plot3D[Sin[Exp[x y]],{x,-2,2},{y,-2,2},PlotPoints->50,AxesLabel->{“x-axis”,“y-axis”,“z-axis”}]

5) ParametricPlot3D[{Sin[t],Cos[t],Sin[Sqrt[2]t]},{t,0,10Pi},PlotPoints->50]

6) ParametricPlot3D[{Sin[u]/Sqrt[v],Cos[2u],v},{u,0,2Pi},{v,1,9}]